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# DSN CONSCAN — A Vector Nomenclature and Method for Determining Parameter Values

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#### I. Introduction

The DSN CONSCAN algorithm can be conveniently expressed in vector notation. A vector system is developed in this article and used to describe a procedure for determining CONSCAN parameter values.

CONSCAN requires the specification of four parameter values for proper operation: the CONSCAN radius, the scan period, the loop phase angle, and CONSCAN gain. These values are manually entered into the antenna pointing subsystem (APS) computer prior to CONSCAN tracking.

The values vary from station to station and are different for different tracking configurations at a single station. There is presently no standard procedure for determining these values. The procedure discussed here can fill that gap more efficiently than previous graphical methods.

In the following sections, only geometrical theory is discussed with no consideration of noise or for optimizing closed-loop operation. Those topics are treated in Ref. 1.

#### II. Theory

CONSCAN causes the antenna pointing vector to describe a small conical scan (hence, the name) around a reference direction. The reference direction is usually moving (e.g., at sideral rate) so that CONSCAN is superimposed on the grosser antenna motion.

During a full revolution about the reference point, CON-SCAN correlates the antenna position with the received signal strength of the RF source. This correlation provides an estimate of the source position. In closed loop operation this estimate is used to automatically correct the antenna reference point, or scan center, after the end of a scan. In open loop operation it is used to inform operators of the source position relative to the scan center.

The chief value of CONSCAN is that it maximizes received signal strength to within about 0.1 dB of the peak value of the antenna gain pattern regardless of errors between mechanical and RF boresight. The main weakness is that variations in signal level due to noise or source fluctuations can cause it to track erratically.

In the following paragraphs, a special coordinate system will be developed, and then the CONSCAN algorithm and its parameters will be discussed.

#### A. Explanation of Coordinates

A DSN antenna is pointed toward an RF source by the time-dependent conputed prediction angles,  $\alpha_p(t)$  and  $\delta_p(t)$ ; the hour-angle is driven by  $\alpha_p(t)$ , and the declination by  $\delta_p(t)$ . (The angles could be in other spherical systems such as azimuth – elevation or x – y without affecting the validity of the following discussion.)

These predictions are insufficiently accurate to maximize the received signal level, particularly at X-band, which has a very small beamwidth. Therefore the antenna is moved, either manually or by CONSCAN, in an attempt to place the RF boresight nearer the signal source.

For any arbitrary coordinates,  $\alpha$  and  $\delta$ , in the near vicinity of the predicted point, the variables

$$\beta_1 = \alpha - \alpha_p$$

$$\beta_2 = \delta - \delta_p$$

define a nearly rectangular coordinate system if the  $\beta$ s are small enough. Then  $\beta_1$  and  $\beta_2$  can be considered the hour-angle (HA) and declination (DEC) components, respectively, of a vector  $\boldsymbol{\beta}$ .

Figure 1 shows the  $\beta$  coordinates as lines of constant  $\beta_1$  and  $\beta_2$  superimposed on a small square area of the angular sphere. The predicted point is at the origin.

Figure 1 also illustrates the notation convention to be used, where the points S, C, and A stand for source, scan center, and antenna position respectively. The subscripts, read from right to left, indicate vector direction so that, for example,  $\beta_{SC}$  indicates the vector from the scan center to the source point, and  $\beta_{CS}$  (=  $-\beta_{SC}$ ) is in the opposite direction. Vector addition follows the normal rules so that  $\beta_S = \beta_{SC} + \beta_C$ .

While the  $\beta$  system is natural for the APS (the numerical values of the  $\beta$  coordinates, if properly scaled, can be used to drive the antenna from the predict point), it is awkward for CONSCAN because of the convergence of hour-angle lines at nonzero declinations. The  $\beta_1$  coordinates are "squashed," and the computed numerical value of an arbitrary  $\beta$  will not correspond to the great circle angle it spans. This effect can be severe at large declination angles.

A natural coordinate system for CONSCAN is defined by the vector  $X = (X_1, X_2)$  where

$$X_{1} = \beta_{1} \cos \delta$$

$$X_{2} = \beta_{2}$$

$$(1)$$

This "unsquashes" the hour-angle component by the factor  $\cos \delta$  where  $\delta$  is the declination at  $\beta (\delta = \delta_p + \beta_2)$ .

Under this transformation, the vectors shown in Fig. 1 are not physically changed (that is, they retain the same graphical directions and lengths), but the numerical component values are different so that the computed lengths correspond to the great circle angles required by CONSCAN.

CONSCAN computations can now be made in the X system and the results transformed into  $\beta$  when it is time to drive the antenna. For compact notation, Eq. (1) can be written

$$X = \mathscr{F}\beta$$
;  $\beta = \mathscr{F}^{-1}X$ 

where

$$\mathscr{F} = \begin{pmatrix} \cos \delta & 0 \\ 0 & 1 \end{pmatrix}; \mathscr{F}^{-1} = \begin{pmatrix} 1/\cos \delta & 0 \\ 0 & 1 \end{pmatrix}$$

The subscript notation will remain the same for both systems.

#### **B. CONSCAN Algorithm and Parameters**

CONSCAN drives the antenna clockwise around the scan center, C, as shown in Fig. 2. The period of revolution is P seconds, where P is one of the four CONSCAN parameters. Since there is a 3-second pause at the end of each scan, P is usually chosen so that P+3 corresponds to an integer number of minutes (typically one or two minutes).

The parameter R is the angular radius of the scan in degrees and is generally chosen so that the antenna power loss, when the source is at a distance R from the antenna, is about 0.1 dB.

The vector  $\mathbf{X}_{AC}$  represents the antenna position relative to C, and has the length R in the direction  $\theta$  as defined in the figure. The source position is represented by S, and  $\mathbf{X}_{SA}$  is the vector from the antenna to the source. The received power depends upon the angular distance of the source from the antenna; i.e., upon the length of  $\mathbf{X}_{SA}$ .

CONSCAN computes points (indicated by "i" subscript), by

$$\mathbf{X}_{ACi} = -R \left( \sin \theta_i(t), \cos \theta_i(t) \right)$$

where the angle  $\theta$  in radians as a function of time is

$$\theta(t) = \frac{2\pi}{P} t$$

Points are generated ten times per second; the antenna is driven by the conversion to  $\beta$  coordinates.

$$\beta_{Ai} = \beta_C + \mathcal{F}^{-1} X_{ACi}$$

After a scan is completed, CONSCAN computes an estimate of the vector from the center to the source,  $\mathbf{X}_{SC}(EST)$ , and (in closed-loop operation) the scan center position is updated by

$$\beta_C(\text{NEW}) = \beta_C(\text{OLD}) + \mathcal{F}^{-1} \mathbf{X}_{SC}(\text{EST})$$

While the antenna is being driven along its circular path, CONSCAN correlates the position with the received signal strength to find  $\mathbf{X}_{SC}(\mathrm{EST})$ . Except for the effect of signal filtering, to be discussed momentarily, the correlation is the discrete summation:

$$X_{SC}(EST) = \frac{2G}{RP} \sum_{i=1}^{N} \mathcal{S}_{i} \hat{X}_{ACi}$$

Instead of the summation, we will use the integral

$$\mathbf{X}_{SC}(EST) = \frac{2G}{RP} \int_{0}^{P} \mathcal{S} \hat{\mathbf{X}}_{AC} dt$$
 (2)

since N, the 10 P points in a scan, is large.

 $\mathscr{S}$  is a voltage (from the receiver AGC circuitry for spacecraft tracking, or from a square law detector for natural celestial sources) that varies with the received signal strength, and  $\widehat{\mathbf{X}}_{AC}$  (= -(sin  $\theta$ , cos  $\theta$ )) is a unit length vector in the direction of  $\mathbf{X}_{AC}$  G is a scaling factor.

The integral part of Eq. (2) is simply a vector that points toward the maximum signal, assuming an axially symmetric antenna pattern with no noise; while the factor 2G/RP scales

the length of this vector. This is illustrated in Fig. 3 where the integral part is represented by I, and the value of  $\mathscr{S}$  is indicated by the width of the band around the scan center. Then it is apparent that the correlation is very similar to finding the center of mass of a circular ring whose mass (analogous to signal strength) varies along the circumference.

The (1/RP) part of the scaling factor provides normalization, keeping the length of  $\mathbf{X}_{SC}(\text{EST})$  approximately constant for different radii and periods, while G is essentially a fudge factor that adjusts the length of  $\mathbf{X}_{SC}(\text{EST})$  to agree with that of  $\mathbf{X}_{SC}(\text{actual})$ . G is called the "CONSCAN gain," and is the third of the four parameters. It is determined empirically, given a particular configuration, and that is the subject of the next section.

Finally, the factor of 2 floating in the scaling factor is rather superfluous given the empirical nature of G, but it is included here for the sake of accuracy. Its origin and purpose are obscure.

Equation (2) would be satisfactory for the CONSCAN correlation except that  $\mathscr S$  is actually a filtered voltage, so that variations of  $\mathscr S$  can lag behind the instantaneous signal level changes by an appreciable amount. (The amplitude is also slightly affected, but we ignore that here.) Since the scan is clockwise, this time lag shows up as a clockwise rotation of I as shown in Fig. 4 where the entire pattern for  $\mathscr S$  is shifted. This effect could be compensated by delaying the values of  $\mathbf X_{AC}$  fed to the integral, but it is easier to just rotate I by an angle that will account for signal lag as well as for other system delays.

Then the corrected version of Eq. (2) should be

$$\mathbf{X}_{SC}(EST) = \frac{2G}{RP} \mathcal{R}_L \int_0^P \mathcal{S} \hat{\mathbf{X}}_{AC} dt$$
 (3)

where  $\mathcal{R}_L$  rotates I by the angle L, and is equivalent to the matrix

$$\mathcal{R}_{L} = \begin{pmatrix} \cos L & \sin L \\ -\sin L & \cos L \end{pmatrix}$$

 $\mathcal{R}_L$  is a clockwise rotation for a positive value of L, so that L must always be negative to be commensurate with the DSN CONSCAN implementation. The last of the CONSCAN parameters, L, is also determined empirically.

The actual CONSCAN implementation uses

$$\mathbf{X}_{SC}(EST) = \frac{2G}{RP} \int_{0}^{P} \mathcal{G} \mathcal{R}_{L} \, \hat{\mathbf{X}}_{AC} \, dt \tag{4}$$

which is equivalent to Eq. (3) since  $\mathcal{R}_L$  is constant with respect to the integration. Equation (4) uses a little more machine time than is necessary, since the rotations are done inside the summation loop rather than outside.

#### C. Finding G and L

Of the four parameters R, P, L, and G, R and P are usually chosen to fit mission and station requirements, and then G and L are determined empirically to optimize tracking. Typically, the values have been found graphically by plotting CONSCAN data for several different scan centers. This section will develop a method for finding G and L using the vector approach.

Equation (3) is rewritten

$$\mathbf{X}_{SC} = G\mathcal{R}_{L} \mathbf{u} \tag{5}$$

where

$$\mathbf{u} = \frac{2}{RP} \int_{0}^{P} \mathcal{S} \hat{\mathbf{X}}_{AC} dt$$
 (5a)

Equation (5) is now considered to be exact (not an estimate as before). It is illustrated in Fig. 5 where it is seen that  $\mathcal{R}_L$  rotates  $\mathbf{u}$ , and G scales the length. The same figure shows the relation

$$X_C = X_S - X_{SC}$$

where  $X_S$  and  $X_{SC}$  are a priori unknown.

Substituting from Eq. (5)

$$\mathbf{X}_{C} = \mathbf{X}_{S} - G \mathcal{R}_{I} \mathbf{u} \tag{6}$$

Now, values for  $\mathbf{u}$  can be output from the APS (on the standard print-out) in open-loop CONSCAN either by using values of G = 1 and L = 0 for the initial setup, or by correcting the printout for any other values. However, Eq. (6) still has four unknowns: the two components of  $\mathbf{X}_S$ , and the desired values for G and L. It is true that  $\mathbf{X}_S$  can be found by doing a manual boresight, but the accuracy is generally poor and the

results are subject to error. Instead, we continue to consider  $\mathbf{X}_S$  unknown.

By successively tracking at two different scan centers, i and j, two vector equations will be available and sufficient for finding the unknowns.

$$(\mathbf{X}_C)_i = (\mathbf{X}_S)_i - G\mathcal{R}_L(\mathbf{u})_i$$

$$(\mathbf{X}_C)_i = (\mathbf{X}_S)_i - G\mathcal{R}_L(\mathbf{u})_i$$

Differencing these equations and considering that the drift of  $X_S$  is negligible between measurements

$$[(\mathbf{X}_{C})_{i} - (\mathbf{X}_{C})_{j}] = -G\mathcal{R}_{L}[(\mathbf{u})_{i} - (\mathbf{u})_{j}]$$

$$\Delta \mathbf{X}_{C} = -G\mathcal{R}_{L}\Delta \mathbf{u}$$

Now it is easy to find G and L from

$$G = \frac{|\Delta \mathbf{X}_C|}{|\Delta \mathbf{u}|}$$

or

$$L = \left[ \tan^{-1} \left( \frac{\Delta u_2}{\Delta u_1} \right) - \tan^{-1} \left( \frac{\Delta X_{C2}}{\Delta X_{C1}} \right) \right] - 180 \text{ deg} \quad (7)$$

where Eq. (7) assumes that the arctangent values are put into the proper quadrants and L is modulo 360 degrees.

If desired,  $X_S$  can also be found by substituting the values for L and G into Eq. (6)

#### III. Test Results at DSS 12

The procedure was used at DSS 12 on 17 May 1979 using the Voyager 1 X-band signal. Parameters were determined for all three Block III AGC bandwidths, using a radius of 0.008 degree (the beamwidth is approximately 0.1 degree) and a time period of 57 seconds.

A manual boresight was accomplished to locate the approximate position of the source. The APS was then put into open-loop CONSCAN with values of G = 10 and L = -23 degrees. The value, G = 10 was used instead of G = 1 in order to provide an extra decimal printout on the APS, which outputs a least digit value of one thousandth of a degree. The value of L = -23 was used to approximate previous results for L.

Four points were chosen for the scan center, all about 3R from the approximate source position. The value of 3R was chosen so that scanning would be on a steeper part of the antenna pattern with consequently less noisy vector outputs than for the flatter portion near the source.

With the manual boresight point established and computationally used as the new origin (or predict point) for convenience, the scan center was moved to the first point, offset from the new origin by  $\beta_C = (24,0)$  thousandths of a degree. Output for several scans was taken, and then the process was repeated for the remaining points at  $\beta_C = (-24,0)$ , (0,24), and (0,-24) successively.

The raw data output from the APS for narrow-band AGC are shown in Table 1, along with the averages and standard deviations. (The first output for each new scan center was eliminated because of errors due to the large antenna movement during the scan.) The averages were then used as the "true" values of  $(\beta'_{SC})_i$ , the initial estimate of the scan center to the source for scan center "i".

These vectors are now converted to X coordinates

$$\mathbf{X}_C = \mathscr{T}\boldsymbol{\beta}_C$$

By comparing Eqs. (3) and (5a),

$$\mathbf{u} = \frac{1}{G'} \mathcal{R}_{-L'} \mathcal{T} \beta'_{SC}$$

where G' and L' are the initial estimates of G and L. The results for the  $(\mathbf{X}_C)_i$  and  $(\mathbf{u})_i$  for narrow-band AGC are shown in Table 2.

The vectors  $\Delta \mathbf{u}$  and  $\Delta \mathbf{X}_C$  were then computed between successive points, along with

$$|\Delta \mathbf{u}|, |\Delta \mathbf{X}_C|, \tan^{-1} \left(\frac{\Delta u_2}{\Delta u_1}\right), \text{ and } \tan^{-1} \left(\frac{\Delta X_{C2}}{\Delta X_{C1}}\right)$$

These values were used to compute G and L for each successive pair of points. The results are in Table 3.

Finally, the values of L and G were averaged. The results for all three AGC bandwidths are given in Table 4. For closed-loop operation, G is usually taken as 0.5 times the open-loop value given so that overshoots due to noise are reduced.

The procedure could have used  $\Delta s$  between all six pairs of points rather than successive pairs. However, drift of the source point can be rapid enough that the difference between, say, points 1 and 4 could lead to erroneous results. In fact, when  $X_S$  was computed using the new parameter values, this drift could be seen and amounted to a total of about 8 thousandths of a degree (0.008 deg) over the one-half hour of data taking for narrow-band AGC. These results are shown in Table 5.

#### **IV. Conclusions**

Although development of the vector nomenclature seems tedious, the final result is worthwhile by making the CON-SCAN algorithm expressible in a neat, consistent package with the two vector components on an equal footing. Vector relations are more easily visualized than their algebraic equivalents. This can pay dividends for future programming of CONSCAN.

The procedure, as tested, worked adequately and should form the basis for a standard procedure. The data gathering process required about one-half hour for each AGC bandwidth including the manual boresight done between bandwidth changes, so that the total time required (including one-half hour for the initial setup) was about two hours. Most of the data reduction was done using a FORTRAN program written for a Modcomp computer.

### **Acknowledgment**

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## Reference

1. Ohlson, J. E., and Reid, M. S., Conical-Scan Tracking with the 64-m-Diameter Antenna at Goldstone, Technical Report 32-1605, Jet Propulsion Laboratory, Pasadena, California, October 1976.

Table 1. Raw data from APS for four scan centers

i	=	1	2	3	4
$(\beta_C)$ i	=	(24, 0)	(-24, 0)	(0, 24)	(0, -24)
(β' <sub>SC</sub> ) i	=	(-58, 5)	(46, 6)	(-2, -53)	(-10, 79)
50	=	(-57, 5)	(53, 0)	(-10, -50)	(-8, 75)
	=	(-53, 6)	(60, 5)	(-7, -48)	(-7, 80)
	=	(-58, 11)	(54, 5)	(-12, -48)	(-11, 69)
	=	(-76, 5)	(55, 0)		
	=	(-61, 12)			
Average	=	(-60.5, 7.3)	(53.6, 3.2)	(-7.8, -49.8)	(-9.0, 75.8)
Std. Dev.	=	(8.0, 3.3)	(5.0, 3.0)	(4.4, 2.4)	(1.8, 5.0)
R	=	0.008 degree			-
P	=	57.0 seconds			
G'	=	10.0			
L'	=	-23.0 degrees			

Table 2. Conversion to X coordinates

$$\begin{aligned} &(\mathbf{X}_{C})\,i \;=\; \mathcal{F}(\boldsymbol{\beta}_{C})\,i \\ &(\mathbf{u})\,i \;=\; \frac{1}{G'}\,\mathcal{R}_{-L}\,,\,\mathcal{F}(\boldsymbol{\beta}_{SC}')\,i \end{aligned}$$

i	=	1	2	3	4
(β <sub>C</sub> ) i	=	(24, 0)	(-24, 0)	(0, 24)	(0, -24)
$(\mathbf{X}_C) i$	=	(22.67, 0)	(-22.67, 0)	(0, 24)	(0, 24)
$(\beta'_{SC})i$	=	(-60.5, 7.3)	(53.6, 3.2)	(-7.8, -49.8)	(-9.0, 75.8)
(m) i	=	(-4.97. 2.90)	(4 78 -1 68)	(-2.63, -4.30)	(2.18, 7.31)

Table 3. Intermediate computations

Points	Δu	$\Delta X_C$	Δu	$ \Delta \mathbf{X}_C $	$\tan^{-1}\left(\frac{\Delta u_2}{\Delta u_1}\right)$	$\tan^{-1}\left(\frac{\Delta X_{C2}}{\Delta X_{C1}}\right)$	G	L
1, 2	(9.75, -4.58)	(-45.3, 0,)	10.8	45.3	- 25.2	180.0	4.2	-25.2
2, 3	(-7.41, -2.62)	(22.7, 24)	7.9	33.0	-160.5	46.6	4.2	-27.1
3,4	(4.81, 11.61)	(0, -48)	12.6	48.0	67.5	- 90.0	3.8	-22.5

Table 4. Final values of G and L by AGC bandwidth

Narrow	Medium	Wide		
$G = 4.1 \pm 0.2$	3.7 ± 0.2	3.7 ± 0.0		
$L = -24.9 \pm 2.3$	-0.1 ± 4.9	-2.5 ± 2.7		

Table 5. Source drift

Point	$\mathbf{x}_{s}$	Time since boresight		
Boresight	(0, 0)	0 min		
1	(-0.7, 2.2)	13		
2	(-2.1, 2.0)	19		
3	(-2.3, 3.6)	24		
4	(-4.5, 6.8)	29		

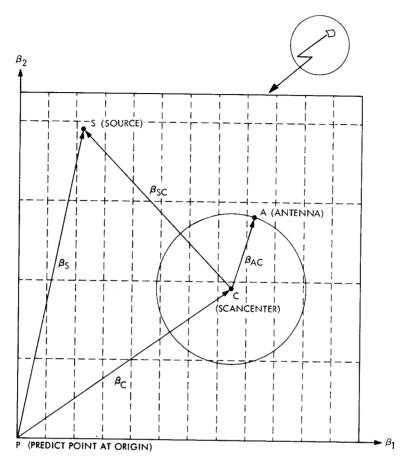


Fig. 1.  $\beta$  coordinate system

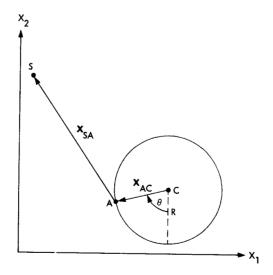


Fig. 2. The scan circle

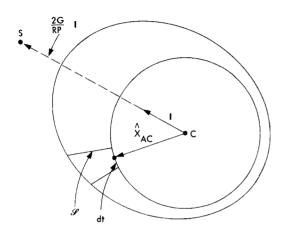


Fig. 3. Signal level variation around the scan circle

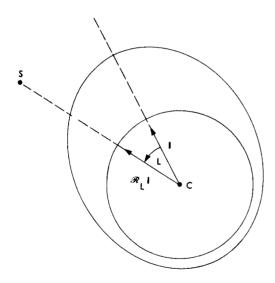


Fig. 4. Effect on filtering on signal level variation

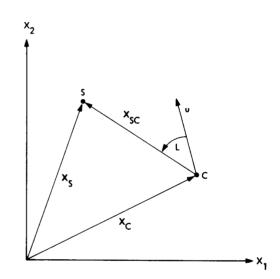


Fig. 5. Vector relationships for determining G and L